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THE INFLUENCE OF INITIAL IMPERFECTION ON THE BUCKLING OF BAMBOO COLUMNS

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ABSTRACT

Increasing the application of bamboo for structural elements requires knowledge about their behaviour under different loading conditions. Only through proved analysis, based on theoretical and experimental results on the performance of bamboo, structural elements, the engineer can accept the responsibility to use bamboo safely and economically in civil constructions. Considering these requirements, since 1979 in Department of Pontificia Universidade of Rio de Janeiro (PUC-Rio) and at a later stage in the School of Engineering of the Federal University of Minas Gerais (UFMG), a series of research programs have been realized to establish the physical and mechanical properties of different bamboo species, connectors for plane and space structures and buckling of bamboo culms. Of great importance for the study of the stability of long elements is the evaluation of the imperfection of the columns' geometry, mainly responsible for the loss of loading capacity in relation to the higher limit, given by the theoretical equation of Euler, which is applied to a column. Bamboo, being a natural product, presents deviations in the column axis with immediate consequences on the loading capacity under compression.

In this article, a device developed to register the geometrical mapping of bamboo column and hence to establish the initial imperfections along the column axis in every two meter length of the bamboo species *Dendrocalamus giganteus*. Further buckling test results on these segments are presented, using the Southwell diagram, which is a classic procedure to understand the global behaviour of the system.

The regularity and consistency of the obtained results showed that the dimensioning of long bamboo elements is viable considering the geometrical variation of the bamboo beside other parameters, which influence the mechanical strength of this natural material. Finally, recommendations are provided for the design of bamboo subjected to compression load.

Keywords: bamboo, columns, geometry, imperfections, buckling tests
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1. INTRODUCTION

About 150 to 200 million years ago a plant appeared on our planet, which today is classified as **bamboo** belonging to the family of gramineas. According to the neurobiologist (Maturana [1]),

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who in the last decades revolutionized the comprehension of vital mechanisms considering all aspects, as an evolution process, consisting in a constant and recurring structural interaction of beings with their surrounding. Although considered to be nothing new, Maturana, however, considers all components of living organisms, such as one's own self, as closed systems, which are determined structurally in such a way that the possible "evolutions" are already foreseen. These would be structural changes with the aim to turn a living being more suitable for its surrounding, which could be described by an observer as a behaviour coordination of a being as a living system, in relation with the environment where this living system is supposed to survive.

From the structural mechanics point of view, bamboo, mainly in order to counteract wind load and the own weight, acquired several natural geometries which turns it a most optimum structure to the requirement of deflection-compression: - a conical form along the culm, an approximately circular transversal section, a hollow form in most species, which reduces its weight, a gradient rigidity to deflection in the radial direction of the bamboo's cross-section, among others.

The appropriation by man, with the aim of utilization in construction, demands, in the same way as in the biological self-evolution, denominated as " autopoiese" by Maturana, that it becomes known to extract the utmost mechanical possibilities of this system already determined, in a way as to establish, with greater precision and safety, the limits of resistance and utilization of the bamboo structure for each type or requirement.

The leap of the bamboo structure is thought to have done in the last decades consisted of passing from an intuitive structural application to a controlled one. As well, an attempt was made to determine to which point it becomes viable to control the structural behaviour of bamboo in its natural state, conforming to the type of requirement and environmental conditions. It is a fact that the intuitive application of bamboo in construction, during millenniums, was and still is extremely useful and without great risks for the users within the limits already established. This attributes to the mechanical control, the task to make it possible to establish a safety index for bamboo constructions, making the structure more economical and making some type of requirements feasible, which so far had been avoided, exactly for involving a greater risk and demanding greater knowledge of the mechanical response.

Therefore, we apply knowledge as a function in order to liberate the evolutionary process of humanity and not as a mechanism of restriction and social-political control, in the way that the mechanical control of the traditional structural systems and other systems ought to facilitate and stimulate the use of this noble plant.

The mechanical behaviour control of bamboo has to pass through its geometrical idealization. For all existing natural forms, man created an idealized form, such as straight lines, circles, ellipses, among others, by which all natural forms would be imperfect manifestations, probably a heritage of the thoughts of the Greek philosopher Plato. As a fact, the so-called perfect forms have their advantages as they facilitate industrialization, the execution and mathematical analysis, but their extreme habitual use could become inconvenient as they make people develop prejudice towards natural forms as being "imperfect". This dis-naturalization of the world, which turns the natural world unreal and the virtual world real, a characteristic behaviour in the actual level of scientific-technological development, brings serious damage to the ecosystems.

Therefore, to draw on these raw materials, trying to integrate them in a symbolic contemporary universe and establishing their form of utilization is as well returning them to the sovereignty of nature and Earth. Thus, with the objective to be able to utilize bamboo, as it is made by nature, in airy structures, such as space structures, geodesics and cable stayed structures in general, we present part of our studies in this paper, (Moreira [2]) which are related to the evaluations of natural deviations in the longitudinal axis and their relevance to the load limit, when the tubular sections are subjected to axial compression.

2. MATERIALS AND METHODS

The first researcher, deducing a mathematical expression for the buckling load of a column, was Leonhard Euler in 1744. For such, all the apparent chaotic phenomena, presenting itself with its many real varieties and relations, was reduced to three relevant variables, adjusted by the square of the irrational number π. One variable represents the rigidity of the material constituent of the column, E, the other represents the geometrical response of the transversal section, when turning around an axis, I, and the last variable is the length of column l. The expression deduced by Euler, $F_E = \frac{\pi^2 EI}{l^2}$, establishes the check critical load F_E, of a long column, which corresponds a theoretical load in which the ideal column is in a straight form or in a slightly curved form.

However, in order to reach this expression, other assumptions have already been made, such as: - the material is homogeneous, isotropic and obeys the Hooke's law; the column is perfectly straight and has a constant transversal section. Also, the column is joined to the neighbouring structural components by means of perfect hinges at the extremity. In the experiments, the Euler load only will be effective to a slender column and the longitudinal axis is perfectly straight one. So, the deviation of the longitudinal axis is a indispensable variable in the understanding of the behaviour of real columns [4].

The study of the behaviour of bamboo under compression leaves us then with some doubts:

- How to measure the initial imperfections of the axis δ_0?
- Does the geometrical approximation of the transversal section in a circular ring form with a constant section throughout the element lead mathematically the necessary results? In which position of the element is it convenient to assume the transversal section to be constant?
- Does the density gradient of the transversal section, in radial direction and from the inside outwards, seen in Figure 1, affect the results?
- How does the failure of the elements occur?

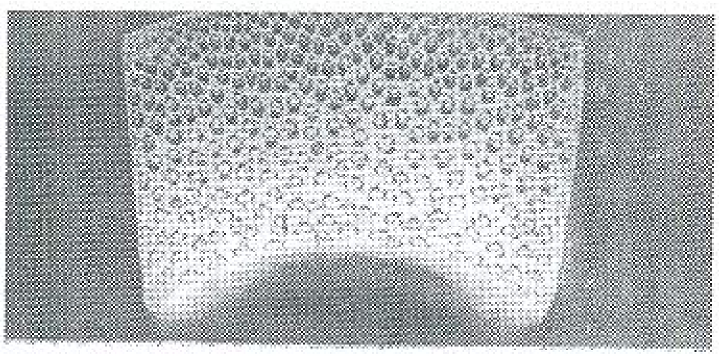

Figure 1 Density gradient

3. MAPPING OF BAMBOO

In order to measure the circumference of the bamboo's geometry with precision, a special mapping devise was designed as shown in Figure 2.

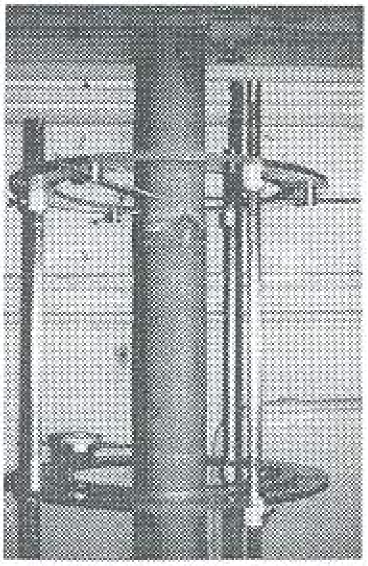

Figure 2 Mapping device developed to measure the Bamboo's geometry

The device consists of three aluminium bars, fixed to a wooden circular base plate and an acrylic ring at the upper end. A second ring is fixed to the upper extremity of the bamboo and a third ring is the mobile measuring ring as shown in Figure 3.

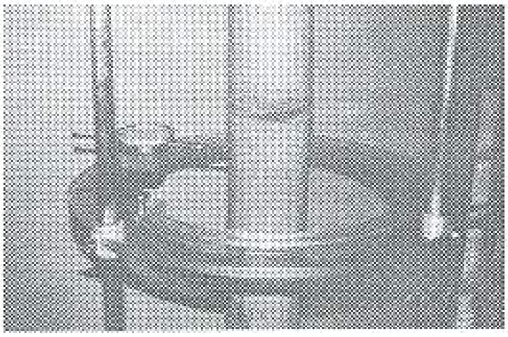

Figure 3 Measuring ring

The measuring ring, consists of an external ring, fixed to the aluminium bars, and an internal ring, which contains a dial gauge with the exactness 1/1000, see Figure 3. This internal ring can turn 360° in steps of 15°. Knowing the distance from the end of the pointer of the dial gauge to the center of the ring, it is possible to reproduce the whole circumference of the bamboo at a determined height. The external circumference of the bamboo is measured at every 10 cm along the axis.

At every 15^0, 24 points were marked on the surface of the bamboo. At each pair of points, a circumferential distance of 90^0 was supposed to belong to one diameter, in a way that the center of this circumference was obtained by the intersection of 2 diameters. After a turn of 15^0, another center could be determined in the same way. The coordinates of these centers were quite close to each other, but did not coincide, because bamboo is not perfectly circular.

It was assumed that the centroid of each circumference was the arithmetic average of the obtained coordinates and the radius was the average distance of this centroid to each of the mapped points on the surface of the bamboo. For bamboo of about 2 *m* length, 19 centroids were determined.

The mapping of the element's axis was obtained by projecting the centroid of each circumference on a plane normal to an imaginary axis which links the circumferential centroids of the two bamboo ends, as shown in Figure 4. Each single mapped bamboo presented a description of the axis different from the others.

After the buckling test, the bamboo culumns was cut in segments of 10 *cm* along the axis and the thickness of the bamboo wall was taken to be the arithmetic average of 4 times the measured thickness at 90^0 distant from each other.

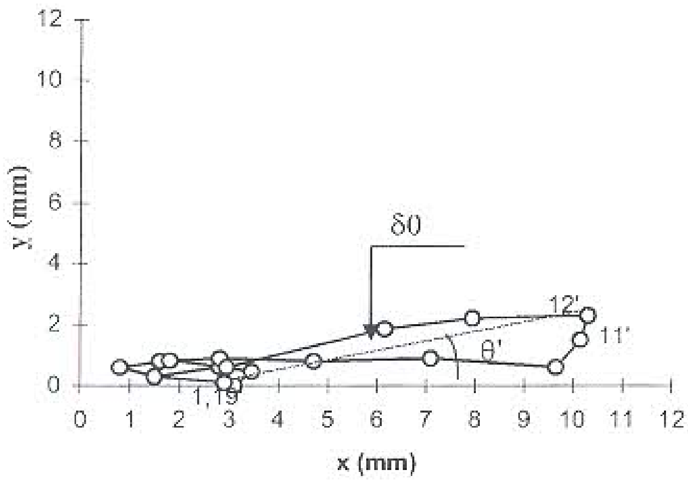

Figure 4 Projection of the centroids

4. DENSITY GRADIENT

The fibres of the *schlerenquima* the part of bamboo's cross-section with high strength, visible as black points in Figure 1, increase their concentration when approaching the external surface. This causes the transversal section to have a density gradient in the radial direction, from the inside to outwards of the thickness, which interferes with the moment of inertia I of the section.

From observations of microstructural studies [1], one considers a gradient function for the radial density: - the transversal section is supposed to have constant density up to closely 75% of the thickness of the internal wall, ρ_i, and a higher density for closely to 25% of more external thickness, ρ_e.

In this way one can calculate for a transversal section, a physical inertia, I_f, larger than the geometrical inertia, or expressed differently

$$I_f = I_{gi} + I_{ge} k_1$$

where $k_1 = \frac{\rho_e}{\rho_i}$, I_{gi} is the geometrical inertia of the internal part ($\approx$75% of wall thickness t) and the geometrical inertia of the external part.

5. BUCKLING

In order to eliminate the initial excentricity in the load application and create hinge ends, the ends of the elements were modeled as shown in Figure 5. Two wooden cylinders were introduced into the hollow ends and glued. In this way the axial load is transferred to the bamboo via a washer, which receives the force through a thread bar, fixed axially. A spherical end of the threaded bar turns inside a spherical opening, and guarantees in this way a hinged condition of the contour for the test specimen.

On the other hand, in order to consider the Euler expression $F_E = \frac{\pi^2 EI}{l_0^2}$, the length l_0 must refer to the rotation axis of the section which is the end of the threaded bar, which adds 5 *cm* to the length of each bamboo specimen. The lateral displacements δ were measured in the centre of the element by means of LVDT and the measurement of the deformations was carried out using electrical extensometers glued along and as well across the element. The LVDT was positioned in direction θ' supplied by the mapping, Figure 4.

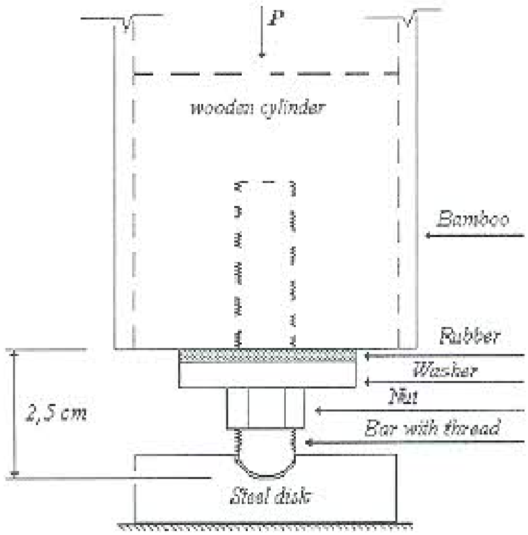

Figure 5 Hinge boundary conditions of the column

6. RESULTS OF THE GEOMETRICAL DESCRIPTION OF THE MAPPING

Eleven elements of about 2m length and about 10cm diameter of specie *Dendrocalamus giganteus* were mapped and tested for buckling. In Figure 4, the extreme circumferences are coincidental, numbers 1≡19 and represent the straight imaginary axis of bamboo. It was possible to detect the section most distant from the axis, obtaining therefore the maximum imperfection δ_0 and the orientation θ' of this maximum imperfection, assuming the preferential direction of the arching of the bamboo. For this reason the LVDT is fixed in this direction.

Figures 6 to 10 show, the type of variation along the element, the average radius R, the average thickness of the wall t, the average area A, as well as the geometrical inertias I_g and physical I_f, of the transversal sections, calculated for each circumferential ring along the axis of the column at a distance of 10 *cm* from each other, respectively.

It can be observed that there are peaks in all curves close to the nodal regions. Further it can be seen that the direction of the radius increase coincides with the direction of the increase of the wall thickness, which is also the direction top-base of the bamboo.

7. MEASURING RESULTS OF THE DENSITY GRADIENT

Bamboo columns of 2 meter length each, were divided in cylindrical segments of 15cm height and in each of these segments $k_1 = \dfrac{\rho_e}{\rho_i}$ was determined and from $k_2 = \dfrac{t_e}{t}$ an average value for all the bamboo was calculated afterwards.

For bamboo with an average density equal to 566±33 kg/m^3, considered to be low, one finds the ratio k_1=1.95±0.18 for k_2=0.28±0.04; now for bamboo with an average density of 883±57 kg/m^3, one finds k_1=1.35±0.09 for k_2=0.27±0.05.

Consequently, for a bamboo with smaller density, the density gradient causes an average increase of inertia for the transversal section, described by ratio I_f/I_g close to 30%. For bamboo with a higher density, this ratio falls to approximately 10%.

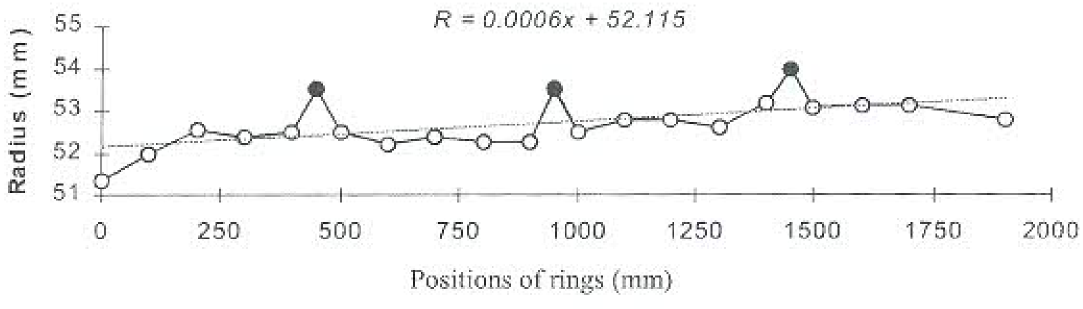

Figure 6 Average radii along the longitudinal axis

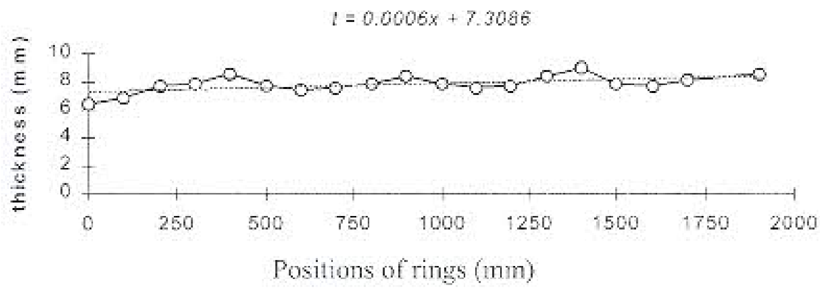

Figure 7 Average wall thickness along longitudinal axis

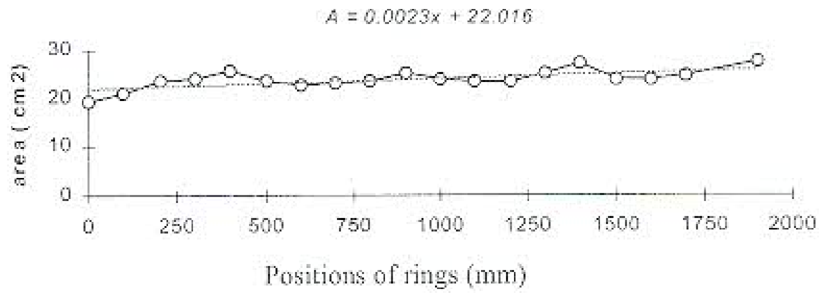

Figure 8 Average area along longitudinal axis

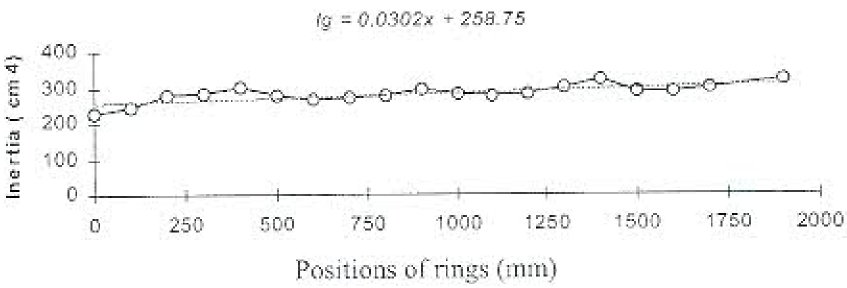

Figure 9 Geometrical inertia along longitudinal axis

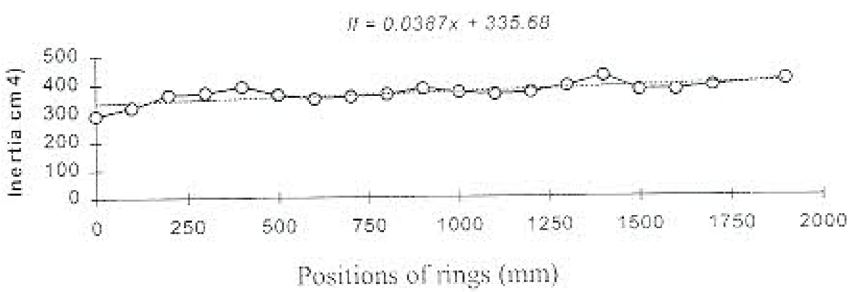

Figure 10 Physical inertia along longitudinal axis

8. RESULTS OF THE BUCKLING TESTS

Table 1 shows the final geometrical characteristics for the studied elements, assuming as the mean value of the characteristics of the extreme transversal sections, which is approximately equal to the arithmetic average of the values obtained from the 19 transversal sections. For each element, the elasticity modulus E was determined. The Euler load F_E was obtained using the Southwell Diagram and P_{lim} was obtained from the test.

Table 1 Geometry of specimens and test results

Test sample	R_e (mm)	t (mm)	I_g (cm^4)	I_f (cm^4)	F_E (kN)	P_{lim} (kN)
5	52.3	7.3	264	303	147.1	46.6
7	45.5	6.2	148	191	60.3	30.7
8	46.0	5.0	128	163	55.0	28.0
9	37.0	4.0	55	76	23.4	16.8
10	42.0	5.1	100	129	30.5	18.2
11	43.5	4.6	99	127	40.8	24.2
12	38.5	6.1	86	109	39.4	16.8
16	49.0	7.2	208	266	78.7	32.6
17	46.0	6.5	161	208	52.6	19.6
18	42.5	6.7	129	167	55.6	19.6
15	52.5	7.9	286	371	95.3	55.9

The instrumentation and the scheme of the buckling test are presented in Figure 11, Figure 12 shows the curves P versus δ_t for the tested bamboo after the total mapping, where δ_t is the total lateral displacement or the sum of the maximum imperfection δ_0 with the deflections measured by the experiment, δ. By plotting the values $\delta/P \times P$, where P is the applied load, one obtains the Southwell diagram for the bamboo, a classic procedure to study the buckling behaviour, independent of the material. Typical failure of the element occurs by progressing squashing of the fibres in the concave region subjected to larger compression stresses. The longitudinal extensometer situated in the most compressed zone, measures the maximum longitudinal strain.

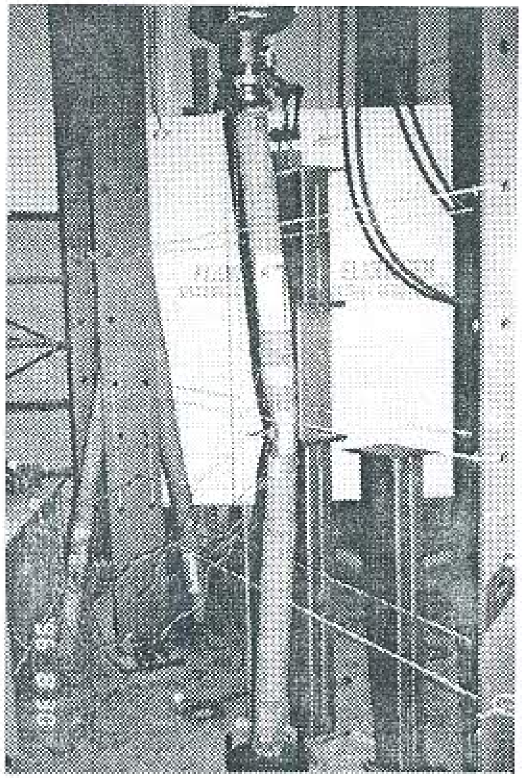

Figure 11 Buckling tests

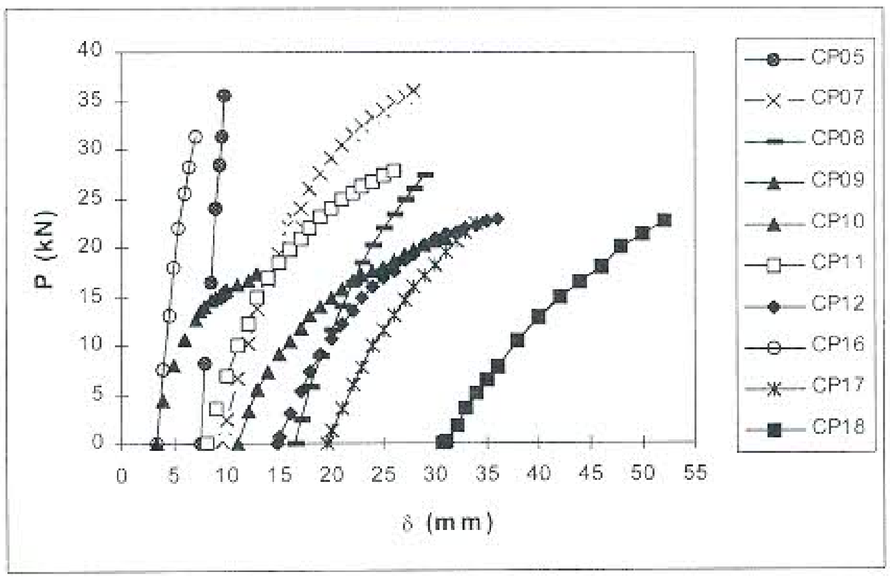

Figure 12 Curves P versus δ_l

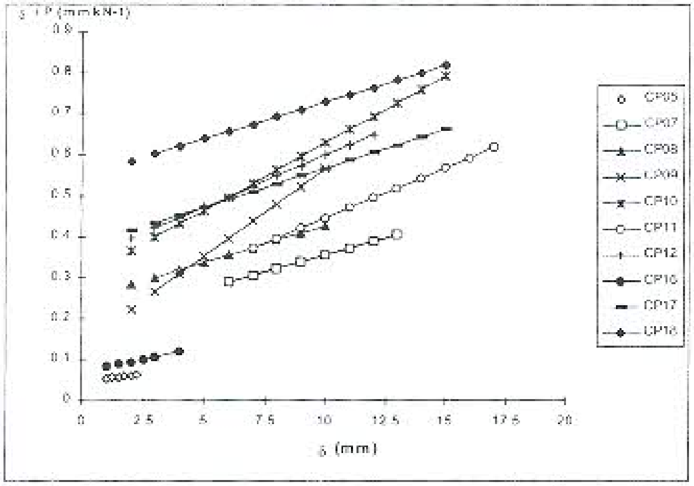

Figure 13 Southwell diagram

9. ANALYSIS AND DISCUSSION OF RESULTS

Theoretically, for a non-prismatic column, the form of the elastic line $y(x)$ must be given by a superposition of sinusoidal half waves, Eq. (1):

$$y = \alpha\left(\frac{\delta_0}{1-\alpha}\sin\frac{\pi x}{l} + \frac{\delta_1}{2^2-\alpha}\sin\frac{2\pi x}{l} + \ldots\right) \tag{1}$$

As $\alpha = P/F_E$ is always smaller than 1 and approaches the unit while P approaches the load Euler F_E, the first term in this series is usually predominant over the others and Therefore one can consider the following approximation:

$$y(x) = \alpha\left(\frac{\delta_0}{1-\alpha}\right)\sin\left(\frac{\pi x}{l_0}\right) \tag{2}$$

For $x = l_0 / 2$, there is a lateral displacement given by

$$\delta = \frac{\delta_0}{\frac{F_E}{P} - 1} \tag{3}$$

By adding δ_0 to the above expression one gets

$$\delta_t = \frac{\delta_0}{1 - \dfrac{P}{F_E}} \tag{4}$$

Rearranging Eq. (3) one can write

$$\frac{F_E}{P}\delta - \delta = \delta_0 \tag{5}$$

Therefore, when plotting $\frac{\delta}{P}$ versu δ one obtains a straight line with an inclination of $\theta = 1/F_E$ which crosses the axis of the abscissa in $\delta=\delta_0$. So, the Southwell diagram, Figure 13 provides the Euler load for the experiment, as well as the initial imperfection of the elements axis. In the following the Table 2 is constructed which compares the imperfections obtained through the mapping δ_0, with the results δ_0' of the Southwell diagram.

The difference between the values δ_0 and δ_0' can be explained by the influence of some parameters of difficult control, such as:

- The presence of small eccentricities in the load application.
- The fact of the Southwell Diagram being considered to be a prismatic bar and the dimensions were taken in l/2 to the way in which bamboo has an variable inertia and the δ_0 measured in the mapping not always occurs in the center of the bamboo.

It is important to point out that the obtained results of the Southwell Diagram represent a global response of the system, or rather, by obtaining the Euler load $F_E = \frac{1}{tg\theta}$, one obtains the rigidity to deflection $(EI)_s$ of the system, which can be expressed with

$$(EI)_s = \frac{F_E l_0^2}{\pi^2} \tag{6}$$

As the value of E_{exp} was determined experimentally for each element, the mean value of inertia of the bamboo can be obtained by $I_s = \frac{(EI)_s}{E_{exp}}$. The comparison of these results with those inertia results obtained by the mapping and density measurements is shown in Table 2.

Table 2 Comparison of mapping results and Southwell diagram

Cp	5	7	8	9	10	11	12	15	16	17	18
$\delta o(mm)$	7.6	9.3	16.5	7.0	11.1	8.2	14.3	7.7	3.7	19.5	30.0
$\delta' o(mm)$	7.0	11.6	13.5	3.3	9.2	8.3	13.6	7.0	5.5	19.9	30.5
If(cm4)	303	191	163	76	129	127	109	354	266	208	167
Is(cm4)	304	170	168	65	136	120	124	373	269	196	153

It can be seen that the density gradient really affects the results, since I_s is more close to I_f than to I_g. The measuring of the maximum strain through electrical strain gauges has shown that the failure of the elements occurs mainly by progressive squashing of the fibres in the concave region of the element followed by the local buckling of the bamboo wall.

The curve σ versus ε for segments of height *2D*, tested in uniform compression, presented the profile of Figure 14:

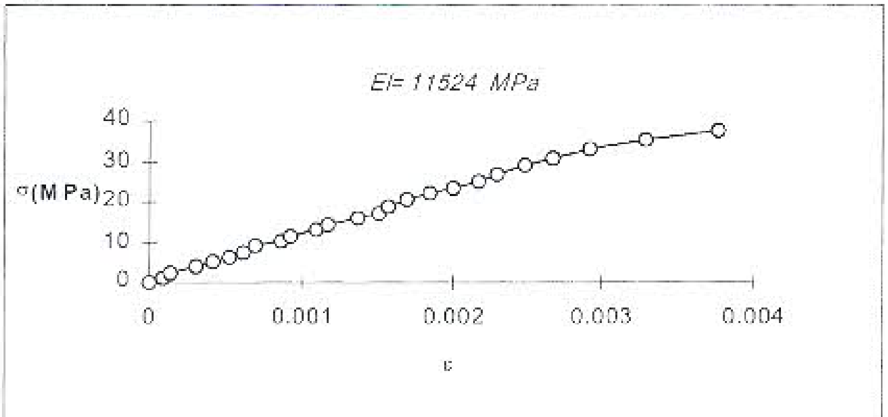

Figure 14 Stress versus Strain curve for bamboo in compression

Therefore, one can assume that the maximum compressive stress responsible for the local failure of bamboo subjected to buckling would be failure stress σ_R. Nevertheless, a more profound study of the measured strain in real time, during the test, in the more compressed zone, longitudinal direction, and comparing it to the theoretical stress acting in the local, obtained through the right side of Eq. (7), points out the limit of proportion σ_p as limit stress corresponding to limit load P_{lim}. This level of stress is the transition between a stable to an unstable behavior of the bamboo wall close to the center of the element, in the concave side where the highest compression stresses occur. It is not so clear if this level of stress may also correspond to instability of the material itself. At the moment of the local buckling phenomenon, the displacement of the bamboo wall was observed as well as the squashing of the fibres. It is not clear what occurp first. Therfore, two limit states should be examined for application of bamboo under axial compression: where is b) ultimate limit state, which guarantees the integrity of the column in compression, expressed by Eq. (7), in which the influence of the axis imperfection is expressed by $\delta_0 / (1\text{-}P/F_E)$.

$$\frac{P}{A} + \frac{P\delta_0 D}{2I_r\left(1 - \dfrac{P}{F_E}\right)} < \sigma_p \qquad (7)$$

The theoretical limit load P_{lim} will occur when the left side of the Eq. 7 equals the right side. Under the calculated limit load, bamboo would have a theoretical lateral deflection given by

$$\delta_t = \frac{\delta_0}{1 - \frac{P_{lim}}{F_E}} \tag{8}$$

The strain limits and the maximum displacements of a structural element, known as service limit states, are mainly associated with the useful life of the materials, aesthetic factors and the comfort of the user. In the case of bamboo, where δ_0 is always present, various test specimens reached maximum loading with the bamboo quite arched, reaching tension stress in the convex side of the element, mainly in those elements in which δ_0 was relatively high.

For the purpose of the project the gain of inertia I_f can be considered negligible and one-can work with $I=I_g$, favorably to safety. Therefore it is possible to solve the problem in an easier way by limiting first the maximum lateral deflection. Thus let $\delta_t=n\delta_0$, and the moment of inertia of the transversal section corresponds to the element approached by $I = \pi\overline{R}^3 t$. Using $F_E = \frac{\pi^2 EI}{l_0^2}$, Eq. (8) can be rearranged ao follows:

$$P_{lim} = \frac{\pi^3 E\overline{R}^3 t(n-1)}{l_0^2 n} \tag{9}$$

This load is shown by P_{lim} because the actions, which load the structural element, are of different combinations to verify the ultimate limit states and service limit states. When an ultimate limit state is reached, due to buckling, the structural element could fail but if the service limit state is reached there is no risk. Therefore, taken as a structural calculus, the calculus actions for verifying the ultimate limit state are always greater than those for verifying the service limit state. Therefore, if P_{lim} represents the limit load of the calculus for verifying the ultimate limit state and P_{lim} represents the limit load for the service limit state, one can write $P_{lim}=\gamma P_{lim}$, where $\gamma>1$ is estimated for each loading. Consequently, once having calculated P_{lim} with Eq.(9), Eq. (10) is evaluated for P_{lim}. It is important to mention that the proposed sequence of the evaluation of these equations is a function of imperfection of the bamboo axis, which is relatively high. However, nothing prevents that the ultimate limit state is first evaluated and then the following service limit state. If the area of the transversal section is approached by $A = 2\pi\overline{R}t$, *Eq.* (7) can be written as:

$$\frac{P_{lim}}{2\pi\overline{R}t} + \frac{P_{lim} n\delta_0}{\pi\overline{R}^2 t} < \sigma_p \tag{10}$$

Acording to the mapping results, the natural form of columns can be selected with a tolerance of $\delta_0 = \frac{l_0}{150}$, where l_0 is the buckling length. This value is twice the initial imperfection considered for cut wooden strips. For three distinct regions of each bamboo specie>column (base, center and top), one can create the parrameter $\lambda_D = D_i/D_e$ sepresenting the relation of internal diameter or radius to external diameter or radius. If

$\overline{R}$ =(R_i + R_e) / 2, the average radius of the center of the element, one can rewrite the Eq. (9) as:

$$P_{lim} = \frac{\pi^3 E R_e^{\,4}(1-\lambda_D^2)(1+\lambda_D)^2}{8 l_0^2}\left(\frac{n-1}{n}\right) \tag{11}$$

In the same way, the Eq. (10) can be rewritten as:

$$\frac{P_{lim}\left[R_e(1+\lambda_D)^2 + 8n\delta_0\right]}{\pi R_e^{\,3}(1-\lambda_D^2)(1+\lambda_D)^2} < \sigma_p \tag{12}$$

10. CONCLUSIONS

Bamboo mapping was basic to the first understanding of bamboo buckling due to the number of relevant variables to the phenomenon. The adopted procedure allowed to determine the buckling plane described by θ', the imperfections characterized by δ'_0 as well as the chaotic distribution of the centroids.

Bamboo tubes in perfect state have behaved as an Euler column with a moment of inertia I_f increased by the density gradient of the fibres in the radial direction. The consideration of the bamboo as a prismatic tube with a constant moment of inertia equal to the mean of the moment of inertia of the ends of the element provides good results for these segments whose maximum mean slenderness is 70.

The limit load of the tested elements P_{lim} were defined by the local instability of bamboo wall close to the center of the element, in the concave side of the bamboo, when the measured strain and estimated stress were both close to ε_p and σ_p, respectively, and not close to ε_y or σ_y given by the stress strain curve of the material bamboo under compression, Figgire 14. In the exact moment of the failure, measured strains have increased indefinitely, resulting in the squashing of the fibres and the displacement of bamboo wall between two adjacent nodes. It was clear that the compression stress that corresponds to this phenomenon is the limit of proportionality σ_p in uniform compression but it was not clear if this level of stress may also correspond to instability of the material bamboo itself.

The proposed equations were fitted to the test results and thus are suitable for evaluating the buckling of the bamboo without splitting. It is also important to remember that these equations are working with nominal values of the loads and resistance of bamboo, i.e. without need to increase the loads or to lower the resistance by safety factors. Hence for the project purpose, it is necessary to add to these equations the suitable safety factors and $I=I_g$ can be used instead of I_f, favorably to safety.

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